The strategic asset allocation of the investment portfolio in a central bank

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Abstract

The balance sheets of central banks (CBs) have changed greatly since the financial crisis, due to the growth of domestic and foreign assets in response to economic and policy developments. As a result, the risks borne by CBs have increased, both quantitatively and qualitatively. Many CBs have begun or resumed dealing with less traditional assets, markets and counterparties that have significantly altered their risk profiles, raising important implications for the risk management of the CBs' investment portfolio. The purpose of this paper is to describe the methodological aspects of a strategic asset allocation (SAA) framework that takes into consideration the perspective of a CB's full balance sheet. Here we refer specifically to a hypothetical Eurosystem CB with no exchange rate policy and whose monetary policy actions reflect its contribution to the setting of monetary and financial conditions to maintain price stability over the relevant horizon. The financial risk profile of such a CB is therefore shaped by risks arising from a policy portfolio resulting from monetary policy operations and the domestic investment portfolio. In this setting, we argue that an effective SAA framework should have three main characteristics: (i) an integrated view of all CB assets and liabilities; (ii) a wide and detailed representation of the investment universe; and (iii) a tailored objective function and constraints. This is a completely different paradigm from the standard view, where foreign reserves and other investments are considered as isolated components of a CB's balance sheet. Under the view put forward in this paper, quantitative tools used to optimise financial portfolios that ignore interdependencies with core policy functions offer only a limited insight on real risks faced by a CB.

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1. Overview

Investment portfolio management in central banks (CBs) has evolved substantially since the global financial crisis. The balance sheets of CBs have changed greatly due to the growth of domestic and foreign assets resulting from CB policy actions taken in response to economic and policy developments. As a result, the risks borne have increased, both quantitatively and qualitatively. Many CBs have begun or returned to dealing with less traditional assets, markets and counterparties that have significantly altered their risk profiles.

These developments have had important implications for the risk management of the CB's investment portfolio. As CBs have become more aware of overall risks on their balance sheet, risk management techniques have evolved in response to growing demands for higher returns, while keeping concessions in terms of liquidity and safety to a minimum. This is often achieved through the embrace of a strategic asset allocation (SAA) process in which key portfolio characteristics are defined.[2]

The purpose of this paper is to describe the methodological aspects of an SAA framework that takes into consideration the perspective of a CB's full balance sheet. Of course, CBs are all different, given their particular mandates and institutional settings, and here we refer specifically to a hypothetical Eurosystem central bank with no exchange rate policy and whose monetary policy actions reflect its contribution to the setting of monetary and financial conditions to maintain price stability over the relevant horizon. The financial risk profile of such a CB is therefore shaped by risks arising from a policy portfolio resulting from monetary policy operations (ie provision of CB money against collateral, outright purchases of securities, bank deposits) and the domestic investment portfolio (ie holdings of foreign exchange reserves, gold and domestic financial assets unrelated to monetary policy).

In this setting, we argue that an effective SAA framework should have three main characteristics: (i) an integrated view of all CB assets and liabilities, (ii) a wide and detailed representation of the investment universe and (iii) a tailored objective function and constraints. This is a completely different paradigm from the standard view, where foreign reserves and other investments are considered as isolated components of a CB's balance sheet. Under the view put forward in this paper, quantitative tools used to optimise financial portfolios that ignore interdependencies with core policy functions offer only a limited insight on real risks faced by a CB. Indeed, this framework is based on the supposition that the primacy of institutional objectives gives rise to unavoidable risks that the investment portfolio can help mitigate.

While this framework is discussed in the context of a hypothetical Eurosystem central bank, it is flexible enough to potentially incorporate features of CBs with different policy objectives and/or economic uses of foreign reserves and domestic portfolios, depending on stakeholder preferences. This wide applicability could be attained, for example, by imposing the necessary constraints in the portfolio construction process (eg through currency and liquidity requirements).

Moreover, given that CBs are exposed to fluctuations in the global economy and financial markets, the determination of the SAA requires estimating returns of a wide range of financial asset classes. The modelling framework should take into account

[2] Borio et al (2008a, 2008b); Bakker and van Herpt (2007).

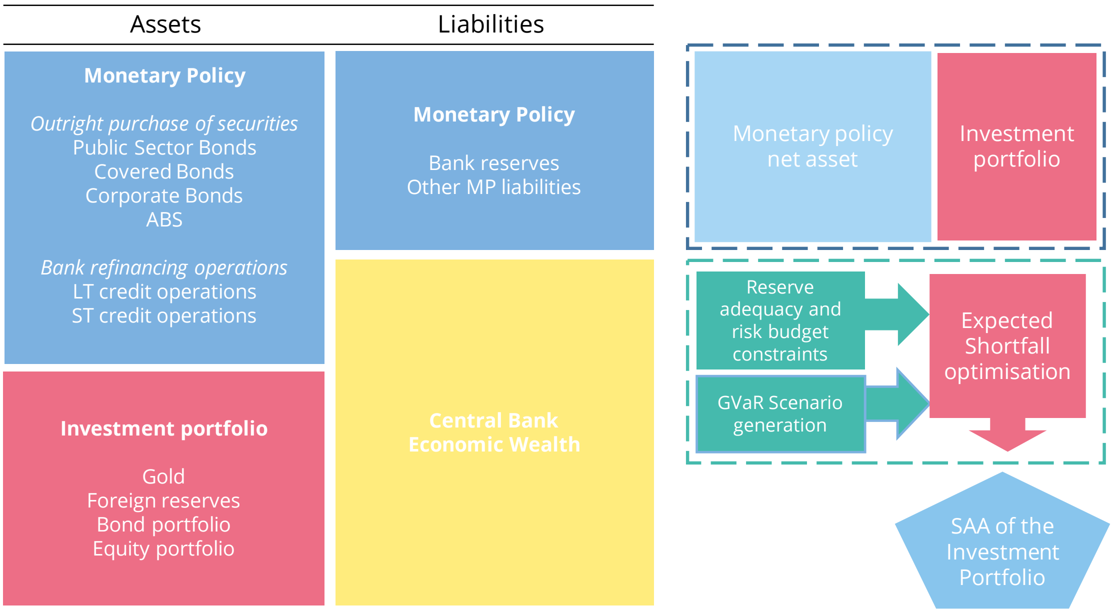
the co-movements that exist across markets and countries regarding the dynamics of output, inflation, foreign exchange rates, interest rates and equity prices. We use a global vector autoregressive (GVAR) model to simulate consistent scenarios for the economic and financial variables needed to evaluate the dynamics of the various components of the CB's balance sheet over the investment horizon.

CBs are also characterised by a peculiar asymmetric utility function: while risk-averse in normal times, they are much less so under extraordinary circumstances, when they must be prepared to take on significant risks. While many private investors might focus on performance in normal times, there is a broad consensus that CBs should be most concerned about portfolio performance during periods of stress. The rationale is that the health of the CB financial structure has to be robust especially in those adverse circumstances in which its institutional functions may lead to taking exceptional risks. It is during these times when, ideally, the investment portfolio should be a source of strength, not a factor that could further jeopardise the CB's financial position. The SAA problem of the investment portfolio is therefore solved by minimising the expected loss in the CB economic wealth (defined as the sum of investment and policy portfolios, both valued at market prices, net of bank reserves and other monetary policy liabilities) over the most adverse scenarios at the end of a long-term investment horizon. The optimisation is subject to a set of short-term constraints, which serve to temper the long-term nature of the SAA and to account for reputational risks. These constraints essentially control the risk associated with accounting losses and depletion of financial capital over a 12-month horizon. The overall scheme of our approach is represented in Figure 1.

Finally, we note that the SAA framework allows some extensions. For example, the CB balance sheet coverage could be broadened to include some implicit items, ie latent assets and/or contingent liabilities, that arise from a CB's policy mandates. These peculiar items are linked to a CB's institutional functions, such as banknote issuance and financial stability, implying an exposure to unavoidable risk factors, with potentially important implications for investment portfolio management and strategies. In this broader setting, economic wealth could be broadened to include the present value of future income from banknote growth (ie the interest costs saved because of the CB's power of issuing such a non-interest-bearing liability) and the contingent liability related to the lender of last resort function of the CB (which may imply the support to illiquid but solvent domestic banks). While these extensions are not explicitly included in the framework presented in this paper, we discuss them as exploratory ideas.

The remainder of the paper is organised as follows. In Section 2, we explain the importance of a CB having an integrated view of all financial risks in a CB's operations in order to define an effective SAA (eg we examine the role of foreign reserves holdings as a tool to hedge financial stability risks). In Section 3, we introduce the GVAR model used for the scenario generation process, and in Section 4 focus on the chosen portfolio optimisation technique. In Section 5, we present an approach for incorporating implicit assets and liabilities in the SAA analysis before concluding in Section 6 with a summary of the stylised characteristics of the optimal SAA under our framework.

Integrating the policy & the investment management functions of a central bank Figure 1



Source: Authors' example.

2. Integrated view

As in the case of other economic agents, the goal of the SAA in a CB is to find an optimal allocation for the investment portfolio, defined as foreign official reserves and domestic financial assets unrelated to monetary policy, across different asset classes in a way that reflects a specific combination of long-term risk-return preferences, general investment constraints and possibly the exposure to unavoidable policy-related risk-return factors.[3] For CBs, important portfolio management implications arise from their unique policy mandates, organisational structure and eligible financial assets. A thorough assessment of these aspects then becomes crucial to provide a rational and consistent basis for determining the SAA.

Bindseil et al (2009) present a comprehensive and conceptually structured framework for the risk management in CBs. The authors explicitly address the need to overcome the widespread practice of segregating CB risk management tools between investment and policy portfolios; they review CB practices in the SAA area and briefly describe the European Central Bank's (ECB) approach based on a highly sophisticated optimisation model. Yet, they refrain from introducing a comprehensive quantitative framework, whereby the SAA is contingent on the core policy function of a CB. More recently, the importance of having a risk management framework that enables a comprehensive and consolidated view of all relevant risks in a CB has been

[3] For example, in the case of private investors, a major exposure to a non-alienable risk-return factor is normally related to their "human wealth", defined as the present discounted value of their future expected labour earnings (Campbell and Viceira (2002)).

highlighted as a cornerstone principle of the IMF's revised *Guidelines for foreign exchange reserve management* (2013).[4]

An example of interrelation between policy functions and SAA: foreign reserves as a tool to hedge financial stability risks

In this section, we review the main theoretical motivations for holding foreign reserves for our hypothetical Eurosystem CB, and summarise an approach recently proposed in the academic literature that could be used to assess reserve adequacy. This adequacy calculation can then be formally integrated into the SAA framework by serving as a constraint in the objective function.

Obstfeld et al (2008) present a potentially useful framework for assessing reserve adequacy for our hypothetical Eurosystem central bank. While many reserve adequacy approaches emphasise negative balance-of-payments shocks (ie capital outflows), which can take place when the purchase of domestic assets by foreigners suddenly stops, Obstfeld et al suggest that similar shocks can also arise when purchases of foreign assets by domestic residents suddenly surge. As an example of this dynamic, some types of banking crisis feature domestic capital flight through a drain of domestic bank deposits, producing a dynamic that is line with the literature on the interaction between banking crises and currency crises (twin crises). In such a case, a flight from domestic bank deposits into foreign exchange puts the domestic banking system and the exchange rate under extreme pressure, and may eventually require the CB to step in as lender of last resort (double drain risk).[5]

The global financial crisis has highlighted the importance for commercial banks, including those in developed economies, to have access to adequate facilities for coping with foreign currency needs, ie foreign reserves held by the CB or through other currency arrangements.[6] In the even broader view taken by Obstfeld et al, the need for more foreign reserves, beyond the trade and the external debt arguments, is mainly ascribed to the overall short-term liabilities of the domestic banking system, proxied by the M2/GDP ratio.[7] The monetary aggregate M2 represents the bulk of the domestic assets that could be easily sold and reallocated in foreign assets by the domestic private sector (double drain scenario).

The approach proposed by Obstfeld et al assessing the adequacy of the amount of official reserves could be integrated in the SAA framework via an explicit lower

4 Item no 36 in the *Revised guidelines for foreign exchange reserve management* (IMF (2013)).

5 Calvo (1996), Calvo and Mendoza (1996) and Calvo (2006).

6 During the crisis, central banks cooperated to set large swap lines to manage a significant US dollar shortage. The credit facilities (currency swap lines) have been granted by the Federal Reserve to the major central banks to provide US dollar-denominated credit to domestic banks under funding pressure (Obstfeld et al (2009)).

7 Obstfeld et al assess reserve adequacy by estimating a panel regression in which the reserves to GDP ratio (expressed in logarithm) is explained by the following variables: (i) the M2/GDP ratio (expressed in logarithm) as a proxy of the size of the banking system and, more generally, of the financial development of the country; (ii) a measure of financial openness, based on a (properly scaled) index, defined by Edwards (2007); (iii) two (mutually exclusive) dummies for the exchange rate flexibility, soft pegging and hard pegging, as suggested by Shambaugh (2004); (iv) a dummy variable for the advanced economies; and (v) a measure of the original sin, ie the inability of a country to borrow abroad in its own currency, proxied by the internationally issued securities issued in foreign currency, as suggested by Eichengreen et al (2005).

bound in the optimisation process for our hypothetical Eurosystem central bank. Of course, alternative measures of reserve adequacy, such as the Guidotti-Greenspan rule that looks to a country's short-term external debt position, could be more appropriate for CBs outside the Eurosystem context.

3. Estimating return distributions for investment and policy portfolio assets

The SAA of the investment portfolio requires estimating the expected return distributions of a potentially wide range of financial asset classes across many currencies. This means that a modelling framework has to be employed that takes into account the interdependencies and co-movements across output, inflation, foreign exchange rates, interest rates, equity prices and any other relevant variables. This section discusses the modelling of asset return distributions across the central bank balance sheet, including foreign (marketable) assets, domestic marketable assets for policy or investment purposes, and domestic non-marketable assets for policy purposes.

The simulation of the financial asset returns

We employ the GVAR model, originally introduced in Pesaran et al (2004) and further developed in Dees et al (2007), to simulate financial asset returns. It is a compact global econometric model that is capable of generating simulations (ie density forecasts) for a core set of macroeconomic and financial variables across a large number of countries in a consistent manner.[8]

Building on the studies of Pesaran et al (2004) and Dees et al, we extend the model along two dimensions: (i) the geographic coverage is broadened to also include Ireland, Greece and Portugal, hence estimating a model with 36 countries (Figure 2); and (ii) corporate spreads and the gold price are added to the financial variables. At the end of 2011, the output from the 36 countries accounted for around 80% of world GDP. The US economy represented one quarter of the total GDP (in power purchase parity) from the 36 countries, followed by the euro area countries (18%), China (16%), Latin America countries (8%, with Brazil and Mexico as the main contributors). Details on the specification and estimation of the model are reported presented in Annex 1.

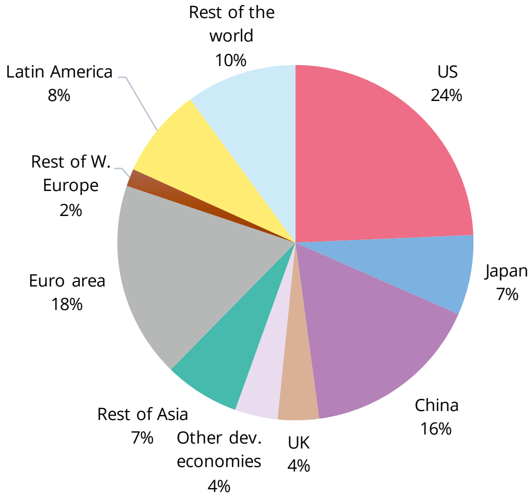
[8] Implementation of this model by practitioners has been supported by the availability of code programmes for estimation and statistical testing. To estimate the model, we used the GVAR toolbox developed by the Centre for Financial Analysis & Policy of Cambridge Judge Business School (University of Cambridge, Smith and Galesi (2011)). We developed our own routines to perform scenario simulations.

Coverage of the GVAR model — Figure 2

List of countries

USA (*)	**Euro area**	**Latin America**
Japan (*)	Austria (*)	Argentina
China (*)	Belgium (*)	Brazil
United Kingdom (*)	France (*)	Chile
	Germany (*)	Mexico
	Italy (*)	Peru
Other dev. economies	Netherlands (*)	
Canada	Finland (*)	
Australia	Greece (*)	
New Zealand	Ireland (*)	
	Portugal (*)	
	Spain (*)	
Rest of Asia	**Rest of w. Europe**	**Rest of the world**
Korea	Norway	Saudi Arabia
Indonesia	Sweden	India
Thailand	Switzerland (*)	South Africa
Phillipines		Turkey
Malaysa		
Singapore		

Share of global GDP (%)**



*Countries considered for the scenario-generating process. ** In USD, PPP-adjusted.

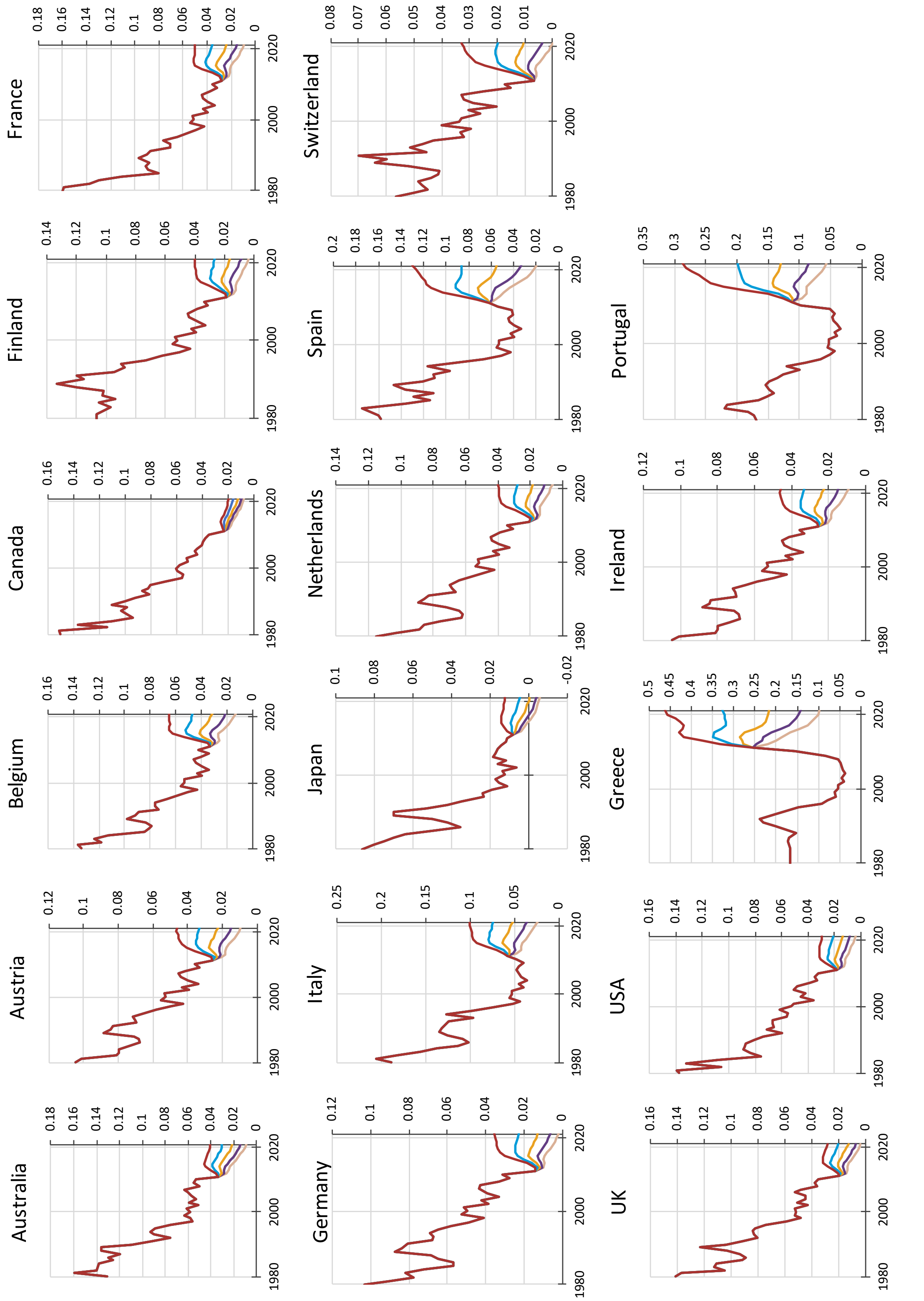
Source: Authors' calculations on World Bank data for 2011.

Our aim is to simulate the evolution of the foreign exchange rates, equity indices, money market interest rates, long-term government yields and corporate spreads needed to calculate the rate of returns for a plausible asset class universe for the investment portfolio of our hypothetical Eurosystem central bank. We also need to simulate the dynamics of the various monetary policy-related components of the economic wealth of the CB. For this purpose, we generate 10,000 scenarios over a 10-year horizon, drawing from the joint distribution of the residuals of the GVAR model.

Figure 3 shows the average and median value, the 75th and 90th percentile calculated on trajectories of 10-year government bond yields for a sample of selected countries. Based on the scenarios generated by the GVAR model, we calculate the financial asset rate of returns.

Yield to maturity on long-term government bonds — Figure 3

In decimal points



Note: Actual values from Q1 1980 to Q3 2012; simulated values from Q4 2012 to Q4 2022: average value, median value, 10, 25, 75 and 90th percentile.

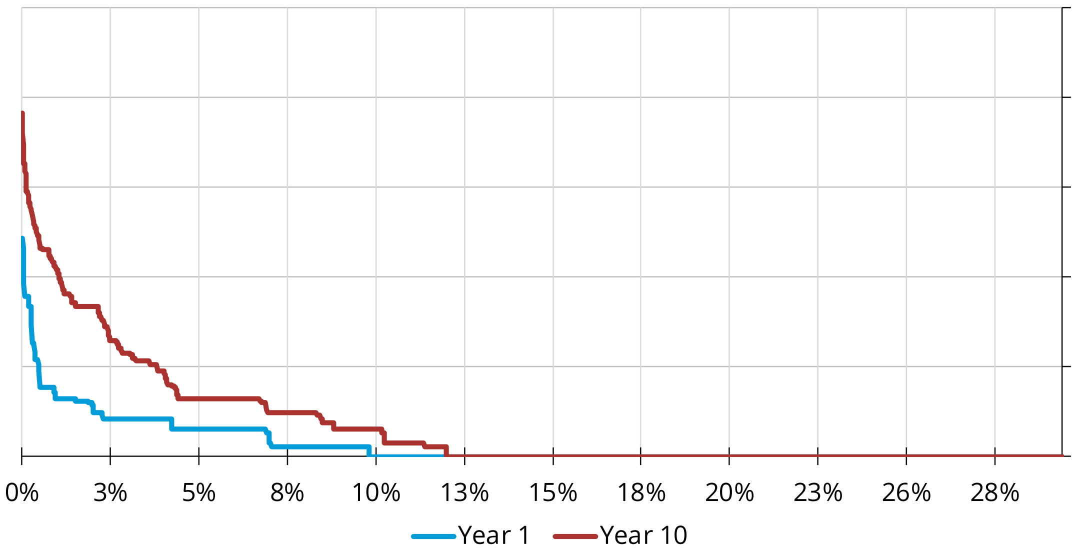
Sources: Authors' calculations.

The simulation of losses on the monetary policy credit operations

The above-mentioned GVAR model covers all the market risk factors that are relevant for the dynamics of the investment portfolio and those marketable domestic assets held for monetary policy purposes. However, in the case of monetary policy credit operations (mainly collateralised loans to banks), market prices are not readily available. In fact, the interest rate earned by the CB is a policy rate that does not reflect the credit risk of the counterparty and the collateral quality (although these aspects are considered carefully in the collateral management framework through eligibility criteria, haircuts, margins and add-ons). Therefore, in order to estimate the loss distribution on the monetary policy credit operations, we directly simulate the event of default for counterparties and collateral issuers (*double-default* model).

For this specific purpose, we take the approach of Pesaran et al (2006), in which an event of default is assumed to occur when the value of firm (counterparty bank and collateral issuer) assets falls below the value of a specific threshold determined on the basis of the credit rating. The dynamics of the value of the firms' assets are simulated through the GVAR. The default distribution at a future point in time, for both counterparties and collateral issuers, is obtained by comparing the simulated asset values with thresholds in different scenarios. Assuming a constant loss given default, the final output is the estimation of portfolio credit losses (Figure 4).

Expected portfolio losses (EPL) at different time horizons and percentile values* — Figure 4



*EPL not disclosed on the vertical axis for confidentiality reasons.

Sources: Authors' calculations.

Further details on the integration of the expected monetary policy credit losses and the scenarios generated within the GVAR framework are reported in Box 1.

Box 1

Credit risk in monetary policy refinancing operations

The approach is aimed at explicitly linking the loss distribution of the monetary policy credit portfolio (that mainly represents a portfolio of collateralised loans to euro area banks) to the evolution of the macroeconomic and financial variables generated by the GVAR model. This setting closely follows Pesaran et al (2006), and is essentially a Merton-type credit model where a firm is expected to default when the value of its assets (V) falls below a threshold value determined by its liabilities (D).

In most empirical applications, the default threshold is taken from balance sheet data (typically short-term debt plus a proportion of long-term debt) while the parameters governing the asset value dynamics are derived by jointly taking into account the default threshold, the market capitalisation of the equity of the firm and the volatility of equity returns. Pesaran et al (2006) apply an alternative, simplified, approach: (i) the default threshold is determined using credit ratings and the equity value dynamics (instead of balance sheet data) and (ii) the firm's asset value dynamics are entirely determined by its equity returns, modelled within the GVAR framework.

Default thresholds are expressed in terms of percentage ratios and indicate the maximum drop that a firm's equity value would be able to sustain before the firm defaults on its obligations. The likelihood of this event is driven largely by the firm's equity returns volatility. We calibrate default thresholds using the long-term historical default frequencies of different rating classes as well as the mean and volatility of equity returns.

We simulate the equity returns for each entity (counterparty and collateral issuer) using (i) equity market returns for each country through the GVAR model (systematic component) and (ii) a firm-specific shock (idiosyncratic component) for each counterparty and collateral issuer.

Defaults occur when the simulated return of the firm's equity value falls below the threshold-equity ratio. Thus, in each scenario, firms will be in a state of "default" or "non-default". Defaults are simulated for both bank counterparties and collateral issuers. Moreover, the collateral issuer state of default or non-default becomes relevant only when the counterparty defaults.

Applying this method to each entity, the default losses are calculated by obtaining the cumulative loss distribution of the credit portfolio over time.

4. Objective function and constraints

While there are no obvious criteria for establishing the appropriate trade-off between risk and return, there is a broad consensus on the desirability of constructing central bank portfolios over a long-term horizon and with a *conservative bias*. In some contrast with private investors, who typically focus on performance in normal times, CBs are most concerned about portfolio performance during periods of stress. This reflects the main objective of a CB's SAA framework: to preserve the value of financial resources required to effectively pursue public policy functions in an independent manner and in any circumstance, that is to say over the long run and especially in

adverse scenarios.[9] This also helps eventually to entrench expectations among both the general public and financial market participants that the CB will not be unduly constrained in the pursuit of its monetary policy and financial stability objectives by concerns over financial resources.

Consequently, a portfolio optimisation approach that aims at minimising risks of losses in adverse scenarios has been preferred to the standard mean-variance framework, where the focus is the minimisation of symmetric risks around an expected outcome. We instead focus on minimising economic wealth over the adverse scenarios.

The objective function of the optimisation

More specifically, the SAA problem of the investment portfolio is solved by minimising the expected loss in the CB economic wealth (investment and policy portfolios, both valued at market prices, net of bank reserves and other monetary policy liabilities) over the most adverse scenarios at the end of a long-term investment horizon. The expected shortfall (ES) metric becomes the natural choice to serve as the primary risk measure under which to construct the SAA.

Although the optimal SAA has a long-term goal, investment policy considerations make it necessary to impose some constraints on portfolio risks in the short term. In this regard, a crucial issue, inevitably faced by CBs, is reputational risk; more specifically, CBs have an above-average sensitivity to capital depletion, to accounting losses and to the range of financial instruments that could be potentially held in the investment portfolio (investment universe). Consequently, the optimisation is subject to a set of short-term constraints that serve to temper the long-term nature of the SAA given these reputational risks. More specifically, these constraints control the risk associated with accounting losses and the depletion of financial capital over a 12-month horizon.

Formally, the SAA problem is specified as a multi-stage stochastic programming model (MSP).[10] There are several reasons why we think the MSP approach is superior to standard mean-variance optimisation (MV). In the first place, the MV framework is based on variance, a measure of risk that cannot accurately handle the non-normal (fat-tailed and/or skewed) return distributions as might be empirically observed, especially for credit risk-sensitive instruments. Moreover, the quadratic utility function used in the MV framework may be inappropriate even with normal distributions, as it counts for positive and negative returns equivalently, while the objective function of a conservative investor should be more sensitive to downside risk. Value-at-risk (VaR) metrics have been widely used as alternative risk measures to overcome this problem, but they suffer from several drawbacks that limit their use in portfolio optimisation applications.[11] These shortcomings are not shared by the ES metric, which can be easily handled within the MSP approach.

9 For two different perspectives on why CBs need to preserve their financial strengths, see Sims (2003) and Cukierman (2011).

10 See Zenios (2007).

11 Since VaR is defined as the expected loss at a given confidence level, it does not give information about the expected losses beyond that confidence level. Losses beyond the VaR threshold may be important especially for non-normal distributions. Theoretical shortcomings of VaR are: (i) it is not

More importantly, the MV framework is essentially myopic, in the sense that the investor groups all returns (which may represent various points in the future) into a single period and identifies an optimal buy-and-hold portfolio for this single period. This strategy simply specifies the proportions of initial wealth invested across the assets in the investment universe and these assets are then held till the end of the investment horizon under all scenarios. The MSP approach offers the significant advantage of considering the investment strategy in a truly dynamic and multi-period context: the portfolio is adapted over time (at specific dates in a discrete time setting) following clearly defined rules. For complex optimisation problems, such as defining the SAA of a CB, MSP may also be viewed as a superior alternative to dynamic programming, which is often associated with the "curse of dimensionality."[12]

The flexibility of the MSP approach may also help to cope with the fact that, under the investment policy of the CB, the management style for certain assets might be more accurately represented by a buy-and-hold strategy. For context, the SAA of a CB is generally defined using the constant-mix rule, where the portfolio is rebalanced annually to the optimal SAA.[13] For example, if the prices of the asset classes in the portfolio change (as would the corresponding weights in the portfolio), the portfolio would be rebalanced by selling (buying) the over (under) weighted ones to re-obtain the SAA mix. The MSP approach offers the advantage of being able to impose the constant mix rule only to a certain part of the portfolio, for example, the ex-gold portfolio.

The constraints of the optimisation

The risk-return preferences of the CB and other investment policy issues have been incorporated in the SAA optimisation process via several constraints:

1. A lower bound for the foreign reserves, including gold, that incorporates the considerations of the reserve adequacy analysis addressed in Section 2;
2. A constraint on the asset weights that aims to keep under control short-term financial risks (*financial* risk constraint);
3. A constraint to control the risk of the portion of the investment portfolio whose accounting valuation is based on market prices (*accounting* risk constraint).

The purpose of constraints 2 and 3, both calculated over a one-year horizon, is to define a risk tolerance for the SAA. The financial constraint requires that the expected losses on the investment and policy portfolios, measured with the ES metric at the 99th percentile over a 12-month horizon (one-year ES 99%), do not exceed the *financial capital allocated to risks* (Figure 5). This is calibrated by subtracting from the financial capital a desired amount that must be preserved to cover extreme risks, ie those arising from events beyond the 99th percentile of the occurrences (unallocated

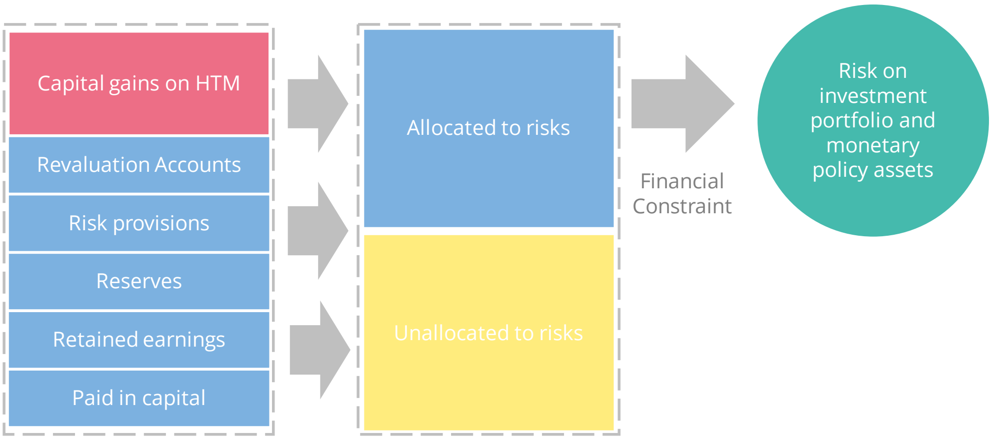
sub-additive (diversification among financial assets may actually increase VaR rather than decrease as conventional portfolio theory would suggest); and (ii) it is, in general, non-convex, which causes great practical difficulties in optimisations applications due to possibly multiple local minima.

12 The dynamic programming framework, both in discrete and continuous time specification, has been of limited practical value for institutional enterprise-wide risk management. The assumptions regarding the utility function and the asset price dynamics are usually restrictive. The framework may even ignore practical constraints such as trading restrictions that are typically imposed by corporate policies and operational requirements (Zenios (2007)).

13 See Zenios (2007) for a discussion of how the constant mix rule is consistent with the logic that, in a multi-stage context, the portfolio decisions must not depend on *clairvoyance* but only on current and past information (CD non-anticipativity constraint).

financial capital). Financial capital is defined as the difference between assets and liabilities valued at market prices or, equivalently, the accounting capital, including the revaluation accounts, plus the capital gains/losses on those assets that are valued at historical cost.

Financial risk control constraint — Figure 5



Source: Authors' example.

As far as the accounting risk constraint is concerned, CBs' typically high sensitivity to reputational risks calls for the minimisation of the probability of a negative result in the annual profit and loss. This constraint is specified by imposing the constraint that the difference between (i) the expected losses (one-year ES 99%) on the investment portfolio assets whose accounting valuation is based on market prices (such as gold, foreign exchange reserves, equities and marketable bond not held to maturity) and (ii) their respective revaluation accounts, must not exceed the general risk provisions.

5. A practical approach for incorporating CB off-balance sheet assets and liabilities

Important peculiarities arise from the unique policy mandates of CBs, fundamentally altering their risk profile relative to private sector institutional investors. Two important aspects include:

1. The contingent liability related to the *lender of last resort* (LoLR) function, ie the risk exposure that may arise in the future from providing liquidity to solvent banks in adverse scenarios;
2. The present value of *future income from banknote growth*, ie the interest saved by the CB on the potential increase of the stock of banknotes, which is an unremunerated and irredeemable liability.

These two peculiar implicit items – both arising from policy mandates – are extremely important for the SAA, as they define an exposure to unavoidable risk factors and influence the characteristics of the CB risk-return preferences. For these reasons, they could be considered as possible extensions to the SAA framework proposed in this paper: the balance sheet coverage – and the concept of the

economic wealth to be used in the optimisation process – could be consequently broadened to include these implicit items, in addition to the policy and the investment portfolios.

A practical approach could be as follows. For each implicit item, its economic value could be captured through a so-called replicating portfolio, ie a combination of asset classes whose marked-to-market value changes closely track those of the implicit item in different scenarios. This approach allows a CB to use portfolio-related metrics typically employed in risk management when integrating these unique risk exposures into the SAA framework.

Contingent liabilities related to the financial stability function

As LoLR to the domestic financial institutions, CBs have a responsibility for the provision of CB money and/or other assistance to solvent entities facing temporary liquidity problems that may lead to an increase in CB money (eg emergency liquidity assistance, or ELA, in the case of our hypothetical Eurosystem central bank). In a fully integrated risk-management framework, the unique risk-return profile potentially arising from these actions should be captured as part of the SAA process. Indeed, this is another reason why the financial stability function should be central to the SAA process for the investment portfolio.

A simple scheme may help to differentiate the emergency actions to support financial stability between: (i) standard lending facilities used for monetary policy operations (available to banks on their own initiative and requiring normal collateral); (ii) the conventional LoLR function of the CB, which would be against a possibly wider range of collateral and require a solvency test; and (iii) measures for likely insolvent banks (guarantees, capital injections etc). Measures under point (iii) fall clearly beyond LoLR support and outside the remit of a CB, whose involvement in financial stability issues is limited to the first two items.

As indicated previously, the risks related to the financial stability function arise exclusively from the LoLR function, whose aim is to reduce the systemic risk of a temporary liquidity crisis of a solvent bank. In these cases, a loss for the CB arises only if all following conditions jointly occur: (i) the illiquid bank becomes insolvent and does not return the full amount borrowed; (ii) the collateral value is less than the CB loan; (iii) the bank liquidation does not allow recovering the full loan amount. In a nutshell, at least in normal conditions, the risks associated with liquidity assistance may be considered close to zero. However, when facing a large bank whose capital strength is highly uncertain and the collateral quality is in the lower part of the acceptability range, risks of incurring into losses are not negligible anymore.

With this perspective in mind, the evaluation of the contingent liability related to expected losses on possible LoLR operations could be undertaken, for example, using the standard tools of contingent claim analysis (CCA). More specifically, one could identify a combination of financial options that would serve as a replicating portfolio, ie a combination of assets whose marked to market value tends to mimic the liability value under different scenarios (ie a financial stability replicating portfolio).[14] This

[14] The risk-free assets component of this portfolio could be modelled as a position in long-term government bonds with low and stable risk premiums, while the assets component could be

replicating portfolio would then be incorporated in the economic wealth portfolio that is the object of the optimisation procedure.[15]

The present value of future income from banknote growth

The CB is the public institution in charge of issuing banknotes, which are a non-interest-bearing and irredeemable liability. The amount of banknotes that the private sector wishes to hold changes over time; in the most popular money-demand functions (ie a Cagan-type function), it is positively correlated with nominal income (because of the increase in total transaction volume) and negatively with interest rates (because of the higher opportunity costs of holding banknotes).[16]

In an integrated SAA framework, we can take into account the present value of the expected stream of implicit revenues – the interest costs saved resulting from the issue of an unremunerated liability – that is associated with the time-varying stock of banknotes in circulation, and the impact on the overall risk profile of the CB.

The estimate of future interest savings poses a number of tricky challenges. First, the dynamics of future banknote growth are highly uncertain, and depend on the money demand function of the private sector and possibly on technological innovation. Second, an interest rate has to be selected to discount the stream of future increases in banknotes. Finally, a CB can legitimately count on this expected stream of revenues only insofar as it is compatible with price stability and the integrity of the monetary system; therefore the estimate has to be constrained to be consistent with the monetary policy objective of the central bank. These complexities are reflected in the wide range of estimates in the literature. For example, with reference to the euro zone, Buiter and Rahbari (2012) have a central estimate of around 20% of GDP (35% in 2015).

Including the present value of future income from banknote growth in an SAA framework also calls for its representation via a replicating portfolio. As in the previous case of the contingent liabilities related to the financial stability function, this would be a combination of asset classes whose mark-to-market value tends to mimic the systematic component of the present value of future income under different scenarios. For this purpose, a *style analysis* (or similar technique) could be employed in order to identify a suitable replicating portfolio.[17] Thereafter, it could be fully incorporated in the economic wealth portfolio.[18]

modelled as a position in corporate bonds. For a general reference on how to use the CCA analysis for these practical purposes see Merton (1977) and Gray, Merton and Bodie (2007).

15 Using the scenario generation model (see Section 3) it would be possible to project the value of the financial stability replicating portfolio (proxy of the CB contingent liability) over a 10-year horizon and calculate its probability distribution at various future points in time.

16 See Cagan (1956).

17 The present value of future income of banknote growth could be expressed in terms of a combination of money market instruments, long-term bonds and equities (Sharpe (1992)).

18 Using the scenario generation model (see Section 3) it would be possible to project the value of this replicating portfolio (proxy of a CB latent asset) over a 10-year horizon and calculate its probability distribution at various points in time into the future.

6. Concluding remarks

In our full balance sheet approach, the optimal SAA of the CB's investment portfolio is constructed in light of its natural exposure to systemic and business cycle risk stemming from its core policy functions. In a sense, the SAA is a minimum risk portfolio adopting a long-run perspective and taking account of all the foreseeable factors that could determine the CB's wealth, including the risks stemming from institutional functions. The rationale is that the CB's financial structure has to be robust, especially in those adverse circumstances in which its institutional functions may lead to taking exceptional risks.

Ceteris paribus, this natural exposure to systemic and business cycle risk is the main reason that should lead a CB to consider countercyclical, low-credit risk hedging assets for inclusion in the SAA. Such a minimum-risk approach tends to produce a conservative SAA providing low returns in normal times. Although this aspect needs to be carefully considered, our view is that such an SAA is consistent with the *conservative bias* that should motivate risk management at CBs, where rewards from high returns carry a lower weight than the costs from reported losses, especially if they come from the management of the investment portfolio.

Moreover, the annual rebalancing rule involves a *contrarian strategy* (selling asset classes that have risen in price and buying those that have declined) which, provided that asset returns mean-revert, should enhance the risk-adjusted performance of the portfolio over the long run.

Finally, in taking a forward-looking and long-term (through-the-cycle) approach with clearly defined portfolio rebalancing rules, the CB's optimal SAA also plays a fundamental role in minimiing the procyclicality of CB portfolio management.[19] The exposure toward countercyclical assets helps reduce selling pressure in time of crisis, when these assets tend to appreciate. According to research by IMF, at the beginning of the crisis in 2008, CB portfolio managers, concerned about increased credit and liquidity risks, lowered their risk exposure in the investment portfolio, especially by reducing the amount of money held in short-term deposits with commercial banks.[20] These reactions were inconsistent with the large volumes of liquidity provided to banking systems through monetary refinancing operations and may also have had unintentional signalling effects to market participants, exacerbating market turmoil. In any case, such procyclical behaviour sharply contrasts with what should be the typical CB conduct during a crisis.

19 Papaioannou et al (2013).

20 According to Pihlman and van der Hoorn (2010), CB reserve managers joined the flight to quality and collectively pulled out more than USD 500 billion dollars of deposits and other investments from the banking sector from December 2007 to March 2009. McCauley and Rigaudy (2011) document how exposure to government-sponsored enterprises was reduced and securities lending programmes were scaled back. Reserve managers' investment in assets under market pressure was also reduced when credit rating downgrades breached the minimum level for inclusion.

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Annex 1 – The specification and estimation of the global vector auto regressive model

This annex describes the approach we used to specify and estimate the global vector auto regressive (GVAR) model. The GVAR is specified by combining 36 country-specific error correction models (ECMs). For each country model, the domestic economic and financial variables are linked to the corresponding country-specific foreign variables, constructed by weighting the domestic variables of the other 35 countries using bilateral trade weights. The GVAR modelling approach provides a general yet practical global modelling framework for the scenario generation process.

Data

Domestic variables

Most country specific models include the following domestic variables:

Real GDP	$y_{it} = ln(GDP_{it} / CPI_{it})$	Inflation rate	$\pi_{it} = ln(CPI_{it} / CPI_{it-1})$
Real equity index	$q_{it} = ln(EQ_{it} / CPI_{it})$	Short-term (ST) interest rate	$\rho^S_{it} = 0.25 \times ln(1 + R^S_{it} /100)$
Long-term (LT) interest rate	$\rho^L_{it} = 0.25 \times ln(1 + R^L_{it} /100)$,	Real exchange rate	$e_{it} = ln(E_{it} / CPI_{it})$
Corporate spread	$cs_{it} = 0.25 \times ln(1 + CS_{it} /100)$		

where GDP_{it} is the nominal gross domestic product, CPI_{it} the consumer price index, EQ_{it} the nominal equity price index, E_{it} the exchange rate in terms of US dollars, R^S_{it} the short rate, and R^L_{it} the long rate of interest, for country *i* during the period *t*. Due to insufficient data availability, and the fact that not all countries have well developed capital markets, not all countries contain the same set of domestic variables.

Country-specific foreign variables

The country-specific foreign variables (starred variables) are obtained by applying a weighting scheme to the foreign variables: for each country *i*, the foreign variable, for example the long-term rate, is obtained by averaging the long rate for all the other economies considered in the model. The weighted average is obtained by taking as weights the flows of imports and exports of the *i*-th country with respect to all other countries.[21] The foreign variables are:

Real GDP	$y_{it}^* = \sum_{j=0}^{N} w_{ij} y_{jt}$	Inflation rate	$\pi_{it}^* = \sum_{j=0}^{N} w_{ij} \pi_{jt}$
Real equity index	$q_{it}^* = \sum_{j=0}^{N} w_{ij} q_{jt}$	ST interest rate	$\rho_{it}^{S*} = \sum_{j=0}^{N} w_{ij} \rho_{jt}^{S}$
LT interest rate	$\rho_{it}^{L*} = \sum_{j=0}^{N} w_{ij} \rho_{jt}^{L}$	Real exchange rate	$e_{it}^* = \sum_{j=0}^{N} w_{ij} e_{jt}$
Corporate spread	$cs_{it}^* = \sum_{j=0}^{N} w_{ij} cs_{jt}$		

where w_{ij} is the share of country *j* in the trade (exports plus imports) of *i*-th country. For a specified time period (eg one year or longer), these data may be collected in a 36x36 matrix, where for each country in a row, the trade shares with respect to all the other countries are displayed in columns. This matrix shows the degree to which a country depends on the remaining ones.

For example, in the case of Italy, about 70% of its international trade is with the first 10 commercial partners. With the exception of China and the United States, all major partners are European countries (euro area countries, the United Kingdom and Switzerland), with the two core economies, France and Germany, representing one third of Italy's total trade. Applying these trade weights to the domestic variables of Italy's trading partners, we obtain the Italian foreign variables.

In addition to domestic and foreign variables, oil and gold prices are considered as exogenous variables (global variables) in each country specific model, except the United States. Following Pesaran et al (2004) and Dees (2007), given the role of the United States in the global economy, the model for this country has been specified as follows: (i) in addition to the usual core variables, oil and gold prices are included as endogenous variables; and (ii) foreign output, inflation rates and exchange rates are the only exogenous variables (interest rates and equity indices are not included).

The GVAR is estimated on quarterly data from Q2 1979 to Q3 2012. A quarterly frequency is considered appropriate for the prediction of long-term returns. Data on monthly frequency contain significant noise; on the other hand, the time series of annual data are not particularly deep for the euro area.[22]

[21] Research on determinants of business cycle co-movements highlights that bilateral trade is one of the most important sources of inter-country business cycle linkages (Glick and Rose (1999)). A possible alternative could be the use of weights based on capital flows; however this information may be of a lower quality and more volatile than weights based on trade data (Dees et al (2007)).

[22] Scherer B (2007, 2008).

The G-VAR model has been estimated using the G-VAR toolbox developed by the Centre for Financial Analysis & Policy of Cambridge Judge Business School (University of Cambridge). The toolbox was originally launched in December 2010 with the release of version 1.0, sponsored by the European Central Bank. Version 1.1, the one we used, was released in July 2011 and is available to download, free of charge, from the Judge Business School website.[23]

Assumptions

For each series, we imposed a mean consistent with the consensus expectations or, in cases in which these are not available, with average historical values. For real GDP, inflation and long-term interest rates, we used the long-run forecasts by Consensus Forecasts, covering a period of 10 years.[24]

For equities, mean returns are obtained assuming a convergence of the price-to-earnings ratios (PE) in different markets to their respective average historical values (taken as equilibrium values).

For short-term rates, we used the estimate for the current year, available from the most recent survey and we assume an unchanged level over the next two years. From the third to the fifth year, we assume a linear convergence to a long-run level of the short rate, obtained as the difference between the consensus forecasts of the 10-year interest rate and the term spread (long rate minus short rate) calculated over the period 1999–2012.

Average values for US and euro area investment grade corporate spreads are taken by forecasts produced by two prominent investment banks.

For the gold price, we imposed a mean real rate of return equals to zero (in US dollars). Historical evidence shows that over the last 200 years (from 1802 to 2003) the gold rate of return has closely followed the US inflation rate.[25] Over the period 2004–12, the price of gold has more than tripled, partly due to the flight to quality observed during the financial crisis.

The specified mean values are consistent with a macroeconomic scenario characterised by a sub-par growth rate, inflation rates and short-term interest rates anchored to policy targets and sovereign spreads (and gold price) consistent with a general normalisation of global economic and financial conditions.

From the GVAR simulations to the asset returns

Based on the scenarios generated by the GVAR model, we calculate financial asset rates of return. For money market instruments, we used the returns on a three-month deposit, for which the interest rate is observed at the beginning of each period. Bond

23 The toolbox is available at http://www-cfap.jbs.cam.ac.uk/research/gvartoolbox/index.html and consists of a set of Matlab procedures executed via an Excel-based interface (Smith and Galesi (2011)).

24 With regard to the yield forecasts for the 10-year government bonds issued by Greece, Ireland, Portugal, Austria, Belgium and Finland, some ad hoc hypothesis have been specified taking into account the spreads of these countries with respect to the yield of the German Bund over the period 1999–2009 (pre-sovereign debt crisis).

25 Over the period 1802–2003, the gold price, deflated by the US inflation rate, provided an overall return of 40%, corresponding to a 0.15% return on an annual basis (Siegel (2004)).

returns are calculated from the simulated long-term yield; the duration of the bond, assuming a constant maturity of 10 years, is calculated via the approximation formula described in Campbell, Lo and McKinlay (1997).

For corporate bonds, yields to maturity are obtained adding the simulated credit spreads to the corresponding government yields. As the typical maturity for corporate bonds is five years, the corresponding five-year government bond yield is obtained from the short-rate time series and the five-year government term spread. This term spread is not directly available in the data set and is obtained by linear interpolation between the short and the long government rate time series. Corporate bond returns are then calculated from the simulated five-year corporate bond yield.

Stocks returns are directly calculated from simulated values of the equity index levels. Bilateral nominal exchange rates for each country in the GVAR model with respect to the US dollar are calculated from real exchange rates and consumer price indices. For euro area countries (EA), nominal exchange rates against the US dollar are weighted by PPP-GDP shares (to total EA GDP). Finally, as we use the euro as *numeraire*, we calculate its value in terms of the other currencies that are considered relevant for investment purposes (UK pound, Swiss franc, Japanese yen, Canadian dollars and Australian dollars).